Integration of 2 Variables

1. Partial Integral wrt x and y – $\int_a^b f(x,y)\,dx$
2. Iterative Integral – $\int_c^d \int_a^b f(x,y)\,dxdy = \int_c^d \left[\int_a^b f(x,y)\,dx \right] dy$
3. Theorem on order – $\int_c^d \int_a^b = \int_a^b \int_c^d$

Example: $\int_{-2}^{0} \int_{-1}^{-2} (x^2+y^2)\,dxdy$

first evaluate inside, $\int_{-1}^{-2} x^2+y^2\,dx = \frac{1}{3}x^3 + y^2 x \Big]_{x=-1}^{x=-2}$

$$= 3y^2 + 3\,dy$$

then calculate the outside, $\boxed{14}$

Example: $\int_3^4 \int_1^2 \frac{1}{(x+y)^2}\,dydx$ first inside $\int_1^2 \frac{1}{(x+y)^2}\,dx = \frac{-1}{x+y}$

then calculate outside, giving $\int_1^2 \frac{1}{(x+3)(x+4)}\,dx = \boxed{2\ln(5) - 3\ln(2) - \ln(3)}$

Example: $\iint_R x\sin y - y\sin x$ first integrate $\int_{x=0}^{\frac{\pi}{2}} x\sin y - y\sin x$

$$= \frac{1}{2}x^2 \sin y + y\cos x \Big]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi^2 \sin y}{8} - y$$

then outside $\int_0^{\frac{\pi}{3}} \frac{\pi^2 \sin y}{8} - y = \boxed{\frac{\pi^2}{144}}$

Example:

23) $\boxed{\text{True}}$

24) $\int_1^4 2x\,dx = \frac{2}{3}x^3 \Big]_1^4 = \frac{2}{3}4^3 - \frac{2}{3}\cdot 1^3$ $\boxed{\text{False}}$

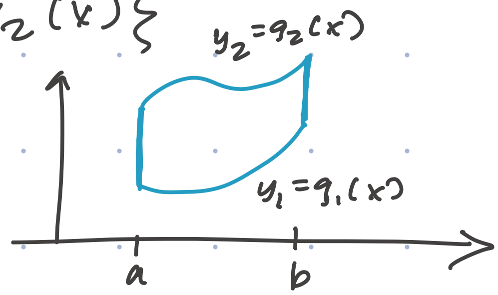
25) $\boxed{\text{True}}$

26) $\boxed{\text{Tue}}$

Double Integral Over Type 1 Region

1. Definition - if there exist a, b, funcs g_1 and g_2 such that $R = \{a \le x \le b \quad g_1(x) \le y \le g_2(x)\}$

① this means

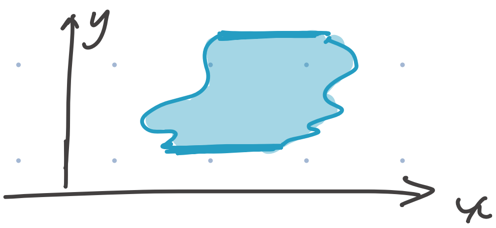


② $$\iint_R f(x,y)\,dA = \underbrace{\int_a^b \int_{g_1(x)}^{g_2(x)}}_{} f(x,y)\,dy\,dx$$

when limit is ~~was~~ function, always <u>inside</u>

Double Integral Over Type 2 Region

1. Same thing as type 1 but its y-axis, like this

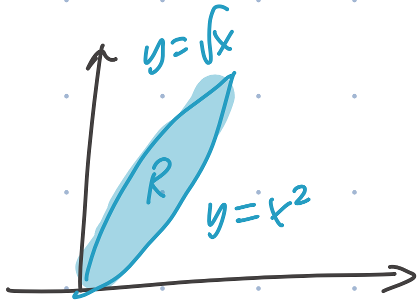


Examples

<u>Example</u>: $\sqrt{x} = x^2$, $\cancel{x} = \cancel{x^4}$ $1 = x^3$, $x = 1$

Type 1 - $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)\,dx\,dy$ interior limits along y

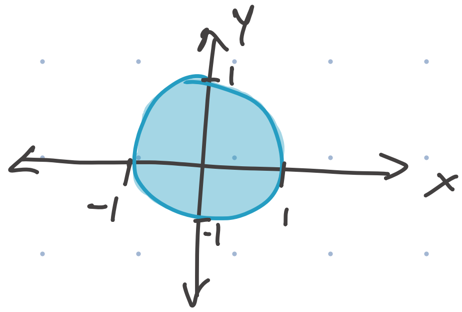
Type 2 - $\int_0^1 \int_{y^2}^{\sqrt{y}} f(x,y)\,dy\,dx$



Example:

Type 1 – $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$

Type 2 – $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

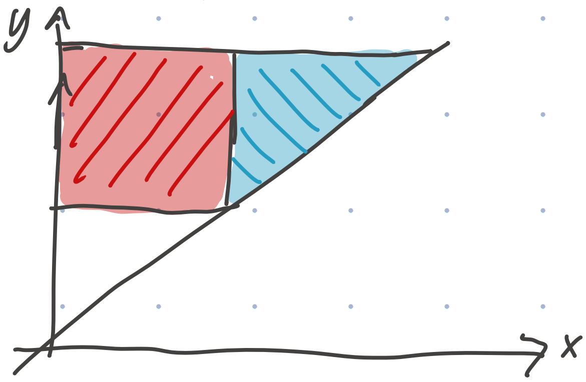


Example:

$\iint_R xy^2\,dA \quad R: \{1 \le y \le 2,\ 0 \le x \le y\}$

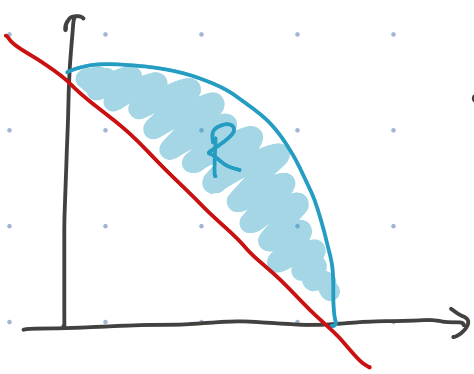
$$\int_1^2\int_0^y xy^2\,dx\,dy = \int_1^2\left[\frac{y^2x^2}{2}\right]_0^y = \int_1^2 \frac{y^4}{2}$$

$$= \left[\frac{y^5}{10}\right]_1^2 = \frac{1}{8}(2)^4 - \frac{1}{8} = \frac{15}{8}$$



$$\int_0^1\int_1^2 y^2x\,dy\,dx + \int_1^2\int_x^2 y^2x\,dy\,dx$$

Example: $\iint_R y\,dA$, R is region in 1st quad $x^2+y^2=25$ and $x+y=5$



$$\int_0^5\int_{\text{interior}}^{\text{exterior}} y\,dy\,dx = \int_0^5\int_{5-x}^{\sqrt{-x+25}} y\,dy\,dx$$

$y=\sqrt{25-x^2}$

$y=-x+5$